



# Use of the model parameter sensitivity analysis for the probabilistic-based seismic assessment of existing buildings

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## Abstract

Seismic assessment of existing buildings is usually treated by international codes and guidelines through a semi-probabilistic approach based on the use of the so-called confidence factor (CF). Many authors revealed the inadequacy of such an approach, proposing alternative procedures based on: the updated calibration of the CF values together with its application to a parameter better representative of the structural response than the material strength, as usually adopted by codes; or the fully probabilistic approach by explicitly considering the propagation of uncertainties. Although the latter constitutes the most rigorous approach, it is still computationally demanding and difficult to be integrated as standard tool in the engineering practice. In this paper, the model parameter sensitivity analysis is proposed to support the seismic assessment in various aspects such as: pointing out, in an explicit way, the influence each uncertain parameter has on the structural response; supporting the set of an effective investigation plan; computing the essential parameters for a probabilistic-based verification on basis of a limited number of analyses. To the latter aim, the results from the model parameter sensitivity analysis executed according to the star design with central point approach are used to determine the median intensity measure ( $IM_{LS}$ ) and, with the help of the surface response technique, its dispersion ( $\beta_{LS}$ ), that are the two parameters of the fragility curve representing the capacity in the assessment. The proposed methodology is applied on two case studies, representative of existing URM buildings. Firstly, the  $IM_{LS}$  and  $\beta_{LS}$  values are calculated and thus compared, for the aim of validation, with the reference ones obtained from nonlinear static analyses performed on a large number of models generated using Monte Carlo simulations. Results obtained show a good estimate of the fragility curve parameters, compared to the rigorous probabilistic approach, highlighting the potential of the procedure proposed.

**Keywords** Model parameter sensitivity analysis · Existing buildings · Probabilistic approach · Response surface technique · Nonlinear static analyses

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## 1 Introduction

The performance-based seismic assessment (PBA) is nowadays the standard approach in the assessment of existing buildings, aiming, as known, to guarantee acceptable levels of risk for the use of the building, the safety of occupants and the conservation of the monument itself, in case of heritage buildings. Posing the latter, many additional controversial issues add up, mainly associated to: (1) overcome the “incomplete” knowledge from which the existing building is intrinsically affected at the beginning of the assessment; (2) interpret and model its seismic response in the most accurate way despite the huge variety of possibilities that characterize it; (3) properly include in the final assessment the residual incomplete knowledge that in general still remain. Issue (1) implies to effectively acquire the as-built information on material parameters and structural details by balancing costs, invasiveness and reliability needs. As far as the issue (2) is concerned, in the assessment of existing buildings, the use of nonlinear analysis approaches (static or dynamic) arises as a very effective tool, especially in case of masonry structures that are the subject of the applications illustrated in this paper. As far as the issue (3) is concerned, it is known that various sources of uncertainties of different natures are involved.

Codes at international or national levels usually face the problem within a semi-probabilistic approach based on the use of a confidence factor (CF). The latter is defined on basis of the knowledge level (KL) reached on the structure under examination through a proper knowledge path and then applied to a given mechanical parameter, assumed a priori as representative to be the most affecting the structural response. As illustrated in Sect. 2, the current CF-based approach presents many deficiencies in guarantying safe results. The alternative could be to pass to a full probabilistic approach that is increasingly emerging not only at research level (Vamvatsikos and Cornell 2002), but also in recommendation documents, as the SAC-FEMA guidelines (Cornell et al. 2002; Jalayer and Cornell 2003) or the CNR-DT 212/2013 (2014) recommendation, recently issued by the Italian National Research Council. Indeed, at research level, various studies analyzed how incorporating the effect of the capacity modelling uncertainty in the seismic assessment through detailed numerical analyses and considering the propagation of uncertainties at various scales (either that of capacity models for structural elements or the global one) and through different probabilistic approaches (see for example Dolsek 2009; Liel et al. 2009; Vamvatsikos 2014; Franchin et al. 2018). Although in principle the full probabilistic approach is the most appropriate in facing all the complex issues involved (Jalayer et al. 2011), many difficulties still endure in its application in engineering practice-oriented procedures. This is due to the huge computational effort and the expertise it usually requires. Indeed, the SAC-FEMA procedure has attempted to address the issue in a convenient way for analysts by converting the probabilistic approach in a semi-probabilistic format through the definition of pre-determined factors representative of building typologies. However, the huge variety of existing buildings makes very difficult the identification of well codified values which are enough versatile to cover all their specificities.

Within this context, the use of the model parameter sensitivity analysis is addressed to improve the aforementioned steps (1) and (3) of the seismic assessment. In this paper, after a short description of the current approaches adopted in codes (Sect. 2), in Sect. 3 it is discussed how the results of a sensitivity analysis “effectively executed” may be useful for addressing the investigation plan and evaluating the basic parameters necessary to proceed with the seismic assessment according to a full probabilistic approach. The main aim of this paper is to provide a solution for the problem of residual incomplete knowledge

included in the final seismic assessment of the building, while the problem of setting an effective investigation plan will be shortly covered being more extensively faced in Haddad et al. (2017). With the aim of pursuing a practice-oriented approach, the assessment is faced by using nonlinear static analyses instead of the more demanding dynamic ones. Then in Sect. 4 the feasibility of the approach proposed is tentatively verified through an application to an URM masonry building. Results achievable by the limited number of analyses performed within the context of the model parameter sensitivity analysis executed according to the star design with central point approach are then compared with fragility curves built from nonlinear static analyses performed on models generated by a Monte Carlo Simulation (Sect. 4.4). The final safety is then checked by mean of the computation of the annual probabilities of occurrence calculated on basis of the closed-form expression presented in Cornell et al. (2002) that is rearranged in a way to propose a safety check more feasible in the engineering practice (Sect. 3.4).

## 2 Current CF-based and full probabilistic approaches

International standards and guidelines (e.g. EC8-part 3 EN1998-3 2005; ASCE/SEI 41-13 2014) treat the topic of seismic PBA of existing buildings by semi-probabilistic procedures, without explicitly taking into account the probabilistic aspect of the problem. Differently from design, that is usually based on linear models and on the adoption for structural parameters of fractiles corrected by proper safety factors, the assessment of existing buildings promotes the use of nonlinear models, which usually refer the use of mean values. The latter needs then to be properly corrected in order to account for the residual incomplete knowledge intrinsically involved in the assessment.

The common approach of standards is based on the definition of a knowledge level (KL), usually divided into three different levels KL1, KL2 and KL3, with increasing achieved knowledge. The attainment of each KL depends on the available data on the structure under examination together with the amount of information acquired on geometry, structural details (or condition assessment in ASCE/SEI 41-13), and material properties. More specifically, for the geometry, the acquisition of the original drawings eventually integrated or completed replaced by a detailed survey is assumed as essential requisite in order to generate an accurate structural model. For material properties, additional information is acquired from both visual inspections and destructive and/or non-destructive experimental tests. EC8-part 3 (EN1998-3 2005) and ASCE/SEI 41-13 (2014) suggest the number of tests and investigations to be performed, without however defining explicitly the locations where investigations should be executed. In this context, the Preliminary Analysis proposed and explained in Haddad et al. (2017) serves as an efficient tool in optimizing such step of assessment.

Concerning the choice of the target KL, EC8-part 3 (EN1998-3 2005) leaves this option free to the engineer or the owner, while ASCE/SEI 41-13 (2014) relates this to the target safety level which one wants to achieve in the ambit of the rehabilitation objectives proposed. For both codes, a pre-defined value of a CF (ranging from 1.35 to 1) then corresponds to each KL that must be applied to one specific parameter, assumed a priori by the code as being the most critical in affecting the structural response. This CF recovers the residual incomplete knowledge, still remaining after investigations, on the parameters used in the final assessment of the structure. In EC8-part 3 (EN1998-3 2005) it is suggested to apply the CF to a mechanical parameter, usually related to strength, while in ASCE/SEI

41-13 (2014) the CF is applied to strength parameters or to deformation capacity, depending on the type of the component. For components classified as deformation-controlled (i.e. showing a ductile behavior), the CF should be applied to the drift limit value, while for components classified as force-controlled (i.e. showing a brittle behavior), the CF should be applied to the mechanical parameters of masonry.

Many authors studied the procedures based on the use of CF (Jalayer et al. 2011; Franchin et al. 2010; Tondelli et al. 2012; Rota et al. 2014; Cattari et al. 2015a) with the aim of investigating the effectiveness and the degree of safety provided by this kind of approach. Some numerical simulations carried on reinforced concrete (Franchin et al. 2010) or masonry structures (Tondelli et al. 2012) to reproduce the results achievable by a large number of virtual analysts proved that sometimes the obtained results are not safe. Moreover, Franchin and Pagnoni (2018) highlighted “how the current code procedure results in uncontrolled and non-uniform fractiles of the corresponding resistance distributions” varying the resistance model considered. The main critical issues that can be singled out by such experiences are: (1) the parameter, which the CF is applied to, is selected a priori although it is not necessarily the one having the highest sensitivity on the structural response; (2) the CF is related to the KL, which is conceptually correct, but its value has no clear justification and the KL reached is defined as the worst among the different examined aspects (geometry, material properties and constructive details) without considering the various effects they may have on the structural response; (3) in case of use of nonlinear analyses (which is recommended in case of existing buildings), the stability of the result from a continuous variation of the assumed relevant parameter is not assured; iv) the pre-defined values of the CF are not explicitly justified in codes and the assumption to achieve the value 1 in the case of the highest knowledge—that means to be able to fully know the structure—appears quite far from reality in practice.

With the aim of overcoming some of these drawbacks, various proposals have been recently outlined in literature. For example, in Rota et al. (2014) it has been proposed to apply the CF directly to the value of the capacity, in terms of the Intensity Measure compatible with the attainment of a given Limit State (LS) ( $IM_{LS}$ ); in this proposal the CF, in addition to consider the acquired knowledge on material properties, includes also other factors accounting for the uncertainty in the modelling assumptions. In a similar manner to the proposal of Rota et al. (2014), in Franchin and Pagnoni (2018) the CF is applied directly to the displacement capacity of the structure accordingly to what it is proposed in the ongoing Eurocodes’ revision work that, differently from the current version, proposes a final value of the KL that derives from the proper combination of various KLs deriving from the knowledge levels achieved on geometry (KLG), construction details (KLD) and material properties (KLM) (Bisch 2018). Moreover, in Franchin and Pagnoni (2018), a specific calibration approach is proposed to assess the CF values associated to the material properties (KLM) that is explicitly linked to the number of measurement of the variable, as a proportion of the maximum number of measurements of the variable (i.e. to the effort of the testing/inspection campaign); the authors showed, through an example focused on reinforced concrete structures, how in this way it would be possible to provide easy-to-use tabulated values for the resistance partial factor for each formula proposed in the Eurocode 8. Finally, a further alternative to face the problem of using pre-defined values of CF has been proposed in Cattari et al. (2015a), where the use of the sensitivity analysis is introduced as standardized tool to calibrate these values and to choose the parameter to which apply the CF, as the one mostly affecting the response without any a priori selection. In the present paper, starting from the work accomplished in Cattari et al. (2015a), another proposal capable to solve the aforementioned problems is illustrated at Sect. 3.

The alternative for including in the assessment the uncertainties treatment in a more robust and rigorous way is to pass to a fully probabilistic approach. This would require the assessment of the fragility curve of each LS, that is usually expressed by a cumulative log-normal distribution described by two main parameters, the median value of  $IM_{LS}$  and the corresponding dispersion  $\beta_{LS}$ , as shown in Eq. (1):

$$p_{LS} = P(d > D_{LS}|im) = P(im_{LS} < im) = \Phi \left( \frac{\log \left( \frac{im}{IM_{LS}} \right)}{\beta_{LS}} \right) \tag{1}$$

where  $d$  is a displacement representative of the building seismic behavior,  $D_{LS}$  is its LS threshold,  $\Phi$  is the standard normal CDF,  $IM_{LS}$  is the median value of the lognormal distribution of the intensity measure  $im_{LS}$  that produces the LS threshold and  $\beta_{LS}$  is the dispersion. In the recommendation document CNR-DT 212/2013 (2014) issued by the National Council of Research, different methods based on the execution of nonlinear Incremental Dynamic or Static Analyses have been proposed to compute the parameters which the fragility curve is based on. However, they require a significant computational effort and expertise, what makes it not yet feasible as the “standard” tool for applications, at least for ordinary existing buildings.

On the other hand, an effective analytical closed-form expression for the computation of the annual probability of occurrence  $p_{LS}$  has been proposed in Jalayer and Cornell (2003):

$$p_{LS} = k_0 (IM_{LS})^{-k} e^{\frac{1}{2} \beta_{LS}^2 k^2} \tag{2}$$

This expression is based on the assumptions that: the hazard function can be approximated by a linear regression on the log–log space (defined by the parameters  $k_0$  and  $k$ ); and the demand and the capacity are independent variables making more feasible the computation of dispersion  $\beta_{LS}$ . The linear regression used to assume the hazard in Eq. (2) presents some drawbacks, as highlighted in Vamvatsikos (2013, 2014) where a second order function has been proposed. Then in Yun et al. (2002), Eq. (2) was also converted in a practical format very effective for applications at engineering level. In particular, similarly to the “load” and “resistance factor” format, it was proposed a probabilistic safety checking developed by replacing the “load” and “resistance” concepts by those associated in seismic field to the “demand” and “capacity”. The latter is shown in Eq. (3):

$$\lambda = \frac{\gamma \cdot \gamma a \cdot D}{\varphi \cdot C} \tag{3}$$

where  $\lambda$  is the confidence parameter,  $\gamma$  is the demand uncertainty factor,  $\gamma a$  is the analysis uncertainty factor related to the one associated with the specific analytical procedure used to estimate the structural demand,  $D$  is the median demand on structure,  $\varphi$  is the resistance factor that accounts for randomness and uncertainty inherent in the prediction of the structural capacity, and  $C$  is the median of the estimated capacity of the structure. The factors which Eqs. (2) and (3) are based on are different but with a similar meaning. Particularly, the factors associated to various sources of uncertainties in Eq. (3) are all combined in  $\beta_{LS}$  of Eq. (2) while the demand  $D$  and capacity  $C$  are represented by the Hazard curve (in terms of  $k_0$  and  $k$ ) and the intensity measure  $IM_{LS}$ , respectively. In Yun et al. (2002), it was also proposed some values for the main parameters involved in Eq. (3). However, studies in

literature are not able to cover the huge variety of features of existing buildings highlighting the inconvenience of adopting reference values representative of a whole class in the context of the assessment of a single building.

To this aim, in Sect. 3 the potential of a limited number of analyses is explored for directly computing  $IM_{LS}$  and  $\beta_{LS}$ , with the main advantage to be “specifically targeted” to the building under examination.

### 3 Potential of the model parameter sensitivity analysis for improving the reliability of seismic assessment of existing buildings

Sensitivity analysis is a technique used in both research and practice areas in order to point out the dependence of an outcome of study from the different sources of uncertainty in the input variables.

In the seismic assessment of existing buildings, the usefulness of this analysis was revealed by various authors (Franchin et al. 2010; Cattari et al. 2015a) and recommendation document (CNR-DT 212/2013 2014) because of its capability to overcome the critical issues mentioned in Sect. 1. In particular, in Cattari et al. (2015a) the sensitivity analysis was explored: to identify the uncertain parameters that mostly affect the response of the structure among all the possible uncertainties; to route the investigation plan and, consequently, deepen the knowledge for some specific parameters only when this is relevant; and to calibrate the value of the CF that should be applied to the parameter identified as the most affecting the response of the structure. To this aim, as proposed in Cattari et al. (2015a), it is helpful to switch from a global scale KL (referred to the whole structure) to different KLs associated to each parameter (or set of parameters) according to its degree of sensitivity since the three aspects defining the structure (materials, construction details and geometry) doesn't affect the structural response always with the same amount.

Differently from the proposal outlined in Cattari et al. (2015a), herein the potential of the model parameter sensitivity analysis is explored to determine, on basis of a limited number of analyses, the two parameters that characterize the fragility curve of the building ( $IM_{LS}$ ,  $\beta_{LS}$ ). The final aim is to include in the safety assessment, in a manner targeted to the building under investigation, the effects associated to the actual variability of the parameters and to the incomplete residual knowledge (represented by the dispersion  $\beta_{LS}$ ). Results of model parameter sensitivity analysis are firstly used to compute the partial dispersions associated to each uncertain variable (Sect. 3.1); then, they are combined to define the total one adopted as dispersion of the fragility curve (Sect. 3.2). Results are validated with a more rigorous probabilistic procedure in order to verify the effectiveness of performing a limited number of analyses: in the case of partial dispersion the target reference is the complete factorial analysis; in the case of the total one the Monte Carlo sampling.

A flowchart of the main steps of the proposed procedure is illustrated in Fig. 1. Steps marked by the light blue are those which the paper is focused on, while additional details on the use of preliminary analysis to completely and effectively address the investigation plan are illustrated in Haddad et al. (2017, 2019).

#### 3.1 Basics to execute the model parameter sensitivity analysis

Firstly, to define the uncertainties involved in the assessment of the building under examination and set the preliminary model adopted for the next numerical analyses, it is essential



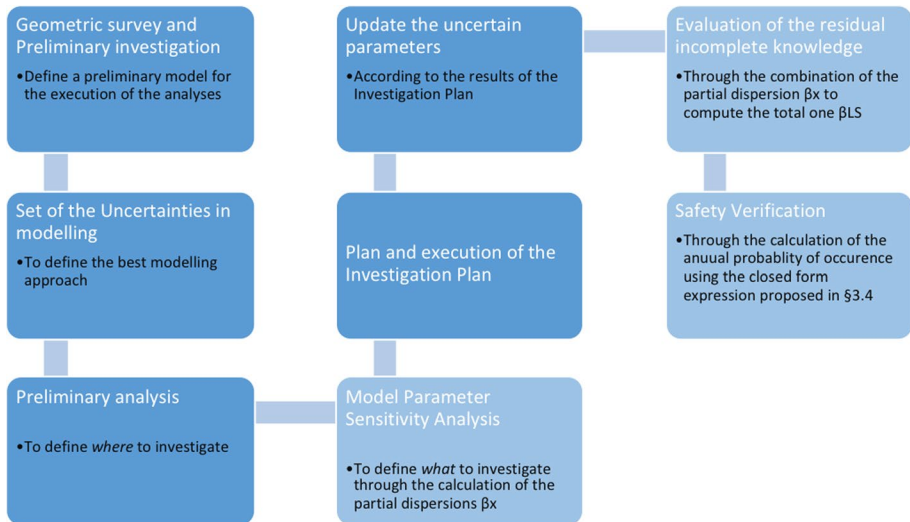


Fig. 1 Flowchart representing the steps of the proposed procedure

to acquire a basic knowledge of the structure. This phase could be accomplished performing a general check of the geometrical data, analyzing the structural details and identifying the main typologies of material that characterize the building. It usually requires just non-destructive investigations, simple essay, virtual inspections and the analysis of literature data.

Uncertainties in the structure may involve various aspects, for example related to: the geometry; the characterization of mechanical properties of materials; the quality of structural details, such as that of wall-to-wall or wall-to-diaphragms connections or that of aseismic devices present in the building (e.g. in the case of tie-rods for which the actual effectiveness could be compromised by the degradation state). Some of these uncertainties can be treated as aleatory variables, i.e. described by a set range of variation and by a probability density function: a typical example is constituted by the mechanical properties of materials. Other ones lead to alternative models of the structure: for example, in case of dubious effectiveness of tie-rods one could decide to consider both options associated to the presence or absence of these elements coupled to masonry. In the latter case, a possible way to proceed in the assessment is using the logic tree approach, attributing to each alternative model considered a subjective probability aimed to quantify its reliability. The sum of the subjective probabilities of each branch is equal to 1 and the safety estimation is obtained through the weighted average of the evaluations performed on each branch.

In this paper, focus will be only on the uncertainties, mostly related to geometry, materials and diaphragms properties, by applying the model parameter sensitivity analysis as proposed. In principle, such sensitivity analysis could be performed to each one of the alternative models eventually considered. Firstly, the engineer is supposed to define for each one of the aleatory variables associated to the uncertainties identified ( $X_k$ ), a plausible range of variation characterized by the median ( $X_{med}$ ), minimum ( $X_{low}$ ) and maximum ( $X_{up}$ ) values. The latter can be determined using information available in codes, literature or previous studies performed on buildings in the vicinity characterized by similar materials. For the strict aim of the sensitivity analysis, it is not required that the minimum and maximum

values are associated to precise fractiles. Parameters could be considered independent or combined into one group assuming that their variation (in terms of lower and upper values) must be identical in each model. Moreover, a more refined correlation law could be assumed within each group of parameters (i.e. when evidences derive from experimental data, for example, in case of stiffness and strength parameters).

The model parameter sensitivity analysis is then executed according to the star design with central point approach by performing  $2N + 1$  numerical analyses (where  $N$  is the number of uncertain variables or groups of variables). Particularly, each one of the  $2N$  models is formed by considering the median values of all the uncertainties but one set once at its lower range value ( $X_{low}$ ) and once at the upper one ( $X_{up}$ ). The additional analysis (+1) is performed using a model with all parameters set at their median values. The execution of a complete factorial analysis would require performing  $2^N$  analyses. It is evident that the number of analyses will increase rapidly by adding more parameters leading to an extremely time consuming numerical effort, even more than a full probabilistic assessment (i.e. faced by the Monte Carlo approach). This is why, in order to balance the computational effort, it is herein proposed to start with a sensitivity analysis comprising only  $2N + 1$  models and then, only if necessary (i.e. as highlighted by the results of such first phase) to add additional targeted analyses, as explained after.

The result of each analysis is here summarized by a Structural Performance Indicator (SPI) represented by the maximum value of the IM compatible with the attainment of a given LS ( $IM_{LS}$ ), which is selected by the engineer to be the best representative of the structural response. In general, for masonry structures that are the object of the case study examined in Sect. 4, a reasonable assumption of the  $IM_{LS}$  may be in many cases the Peak Ground Acceleration (PGA): such approximation (e.g. instead of the use of spectral ordinate associated to the fundamental period) is justified by the fact that they are characterized by a period of vibration rather low. This quantity can be calculated using nonlinear static procedures based on the use of overdamped or inelastic spectra, of which the reliability has been recently discussed in Marino et al. (2018).

### 3.2 Computation of partial dispersions to address the investigation plan

The values of  $im_{LS}$  collected from the model parameter sensitivity analysis are firstly used to define the partial dispersions  $\beta_x$  that reflect the sensitivity of each aleatory variable on the structural response, by capturing the variability of the IM when moving from  $X_{low}$  to  $X_{up}$  of a certain parameter. They can be considered as the angular coefficient of the hyperplane that fits the response surface of the variable  $\log(im_{LS})$  in the hyperspace of the normalized variables representing the aleatory variables. They are calculated using the Response Surface Technique as discussed in Pinto et al. (2004), by using the full factorial analysis through Eq. (4):

$$\beta_x[x = 1 \dots N] = (Z^T Z)^{-1} Z^T Y \quad (4)$$

where  $Z$  is the matrix ( $2^N \times N$ ) of the normalized aleatory variables, with values equal to  $-1$  or  $+1$  (corresponding to  $X_{low}$  and  $X_{up}$  values), in order to consider all vertexes of the hypercube, and  $Y$  is the vector ( $2^N \times 1$ ) of the  $\log(im_{LS})$  quantities deriving from the analysis performed on the models defined by the  $2^N$  combinations.

In the simplified case of executing the  $2N + 1$  analyses, according to the star design with central point approach, the regression is made independently for each variable by



considering the corresponding two interval extremities (normalized variable equal to  $-1$  or  $+1$ ). The partial dispersions are provided by Eq. (4) by adopting a proper Z matrix ( $2N \times N$ ).

The partial dispersions are useful to direct the investigation plan aiming to deepen the knowledge of the structure. In fact, this step allows the identification of the most relevant investigations, focusing on the parameters that produce a great uncertainty in the safety assessment, that are those having a high value of  $\beta x$ .

In some cases, the variation between  $im_{LS}$  of the two analyses performed with the limits of a certain uncertain parameter could be not monotonic, showing values that are at the same side of the one obtained using the median value of such parameter. In this case, it would be better to run additional analyses by varying the values of the other parameters (previously set at the median value) in order to form a clearer idea about the dependence between the value used for the uncertain parameter and the corresponding  $im_{LS}$  and then calculate  $\beta x$ .

The results of the investigations carried out can lead to confirm or update the median values  $X_{med}$  of the intervals of variation of the aleatory variables (assuming implicitly, without the need of a direct estimation, that due to the investigations executed, the initial interval will be reduced to some extent). A practical way to update the median values and the ranges of variation of the aleatory variables could be the Bayesian approach as demonstrated in Jalayer et al. (2011) and Bracchi et al. (2016). As for the modelling uncertainties treated by the logic tree approach, the additional investigations help to acquire information useful in choosing the most reliable model among the alternatives originally assumed, or at least assign to each model a subjective weight, representative of the reliability of each choice.

As mentioned earlier, the setting of the investigation plan will not be further deepened in this paper and the computation of the partial dispersions precludes herein only to find out the total ones as it will be shown in Sect. 3.3. Additional details on the investigation plan and the combined use of a preliminary analysis together with the sensitivity one to assess, not only “what” to investigate, but also “where”, are provided in Haddad et al. (2017, 2019).

### 3.3 Computation of the total dispersion to pass to a full probabilistic assessment

Assuming that it is impossible to reach a complete knowledge of the whole structure, even after investigations, some residual incomplete knowledge will remain and the  $IM_{LS}$  value obtained from the updated median values of the uncertain parameter cannot completely reflect the real capacity of the structure. The results of the model parameter sensitivity analysis could provide a good estimate of  $IM_{LS}$  and  $\beta_{LS}$  that takes into consideration such residual incomplete knowledge. After executing the investigation plan and updating the median values and the intervals of variation of the parameters with high sensitivity, two possible alternatives of proceeding may arise: (1) in the case the median value of any parameter is significantly modified, the sensitivity analysis should be rerun by adopting the modified values for the new models; (2) in the case only the rational intervals already assumed are significantly modified, but the median values remain the same, it is necessary to rerun only the analyses where it was used  $X_{min}$  or  $X_{up}$  of the updated parameter. Thus, the worst case is to rerun  $2N + 1$  analyses again, resulting at the end  $4N + 2$  nonlinear static analyses, which is still considered a low number compared to a full probabilistic procedure.

The new results of the sensitivity analysis are used to define the median value  $IM_{LS}$  of all the  $im_{LS}$  calculated at each analysis performed with the updated variables to proceed to the

final assessment. On the other hand, by reapplying the response surface technique on the logarithm of the new  $im_{LS}$  quantities, it is possible to define the updated Partial Dispersions  $\beta_x$  and the total one  $\beta_{LS}$  of the fragility curve representative of each LS as shown in Eq. (5).

$$\beta_{LS} = \sqrt{\beta_x^T \beta_x} \tag{5}$$

It is worth highlighting one again that the value of the  $\beta_{LS}$  so computed accounts only for the uncertainty in the model parameters. Indeed, it would be easy to include also the effect associated to the record-to-record variability ( $\beta_{LS,D}$ ). The latter could be computed considering not only the median response spectrum in the assessment but also its fractiles at 16% and 84% through the following equation:

$$\beta_{LS,D} = 0.5 \left| \ln(im_{LS,D84}) - \ln(im_{LS,D16}) \right| \tag{6}$$

where  $im_{LS,D84}$  and  $im_{LS,D16}$  are the values of the intensity measure obtained by comparing the capacity, obtained by setting in the model all the variables to their median value, with the response spectra associated to the 84% and 16% fractiles, respectively.

Finally, by assuming that these two contributions are statistically independent, a final value of the dispersion could be computed as well as:

$$\beta_{LS,tot} = \sqrt{\beta_{LS}^2 + \beta_{LS,D}^2} \tag{7}$$

### 3.4 Practice oriented proposal for a probabilistic-based verification

The final safety assessment, in probabilistic approaches, is performed through the calculation of the annual probability of occurrence by combining the fragility curve, representing the capacity of the structure, and the hazard function, representing the possible seismic actions in the region where the structure is located. By referring to the SAC-FEMA closed form expression (Jalayer and Cornell 2003) relating the two aforementioned functions in a practical way, this problem is approached from an easier point of view.

At the same time, in order to put the problem of the seismic assessment of existing structures within a practice-oriented framework, and to make the concept of probabilistic-based assessment easier and simpler to be applied at engineering field, it is convenient to express the result in terms of a value of the intensity measure  $IM^*$  that assures the same probability of occurrence obtained by considering the residual incomplete knowledge, as explained in Eq. (8).

$$\lambda_H(IM^*) = \lambda_H(IM_{m,LS}) e^{\frac{1}{2} \beta_{LS}^2 k^2} \tag{8}$$

Indeed, it is obtained by simply rearranging the formulation of the SAC-FEMA as in Eq. (2).

By referring to the hazard curve in the form of Eq. (9), the probability of occurrence  $\lambda_H$  could be expressed as in Eq. (8), where  $k_0$  and  $k$  are related to the specific site.

$$\lambda_H(IM) = k_0 IM^{-k} \tag{9}$$

$$k_0 (IM^*)^{-k} = k_0 (IM_{m,LS})^{-k} e^{\frac{1}{2} \beta_{LS}^2 k^2} \tag{10}$$



Furthermore, by rearranging Eq. (10), it is possible to express the results in terms of the values  $IM^*$  as shown in Eq. (11), by applying a Confidence Factor  $CF^*$  computed by the obtained value of  $\beta_{LS}$  to take properly into consideration the effect of the residual incomplete knowledge.

$$IM^* = IM_{LS} e^{-\frac{1}{2}\beta_{LS}^2 k} = \frac{IM_{LS}}{CF^*} \quad (11)$$

$$CF^* = e^{\frac{1}{2}\beta_{LS}^2 k} \quad (12)$$

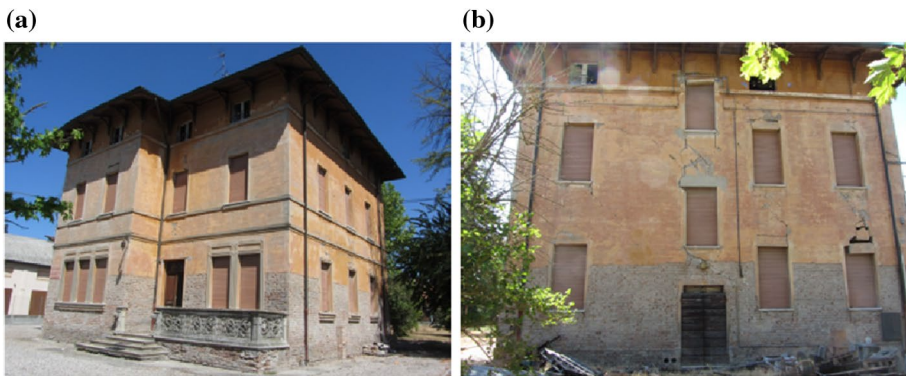
It is worthy highlighting that the confidence factor  $CF^*$  so resulting is specifically targeted to the building under investigation and not conventionally fixed a priori as usually proposed in codes (Sect. 2), representing one of main potential of the procedure herein proposed. This format appears to the analysts like deterministic, making it very convenient for practice-oriented applications.

It is useful to explain why in Eq. (10) and following, the dispersion  $\beta_{LS}$  instead of  $\beta_{LT,tot}$  is used. Indeed, in the safety assessment currently proposed by codes the effect consequent to the record-to-record variability is neglected at all by proposing to use just one response spectrum (in general that representative of the mean value). Thus, to be consistent with such format, only the uncertainty inherent the model parameters has been considered, being very easy in possible future applications to include both terms.

## 4 Application of the procedure

### 4.1 Description of the cases of study

For testing the effectiveness of the model parameter sensitivity analysis and the proposed procedure, a first case study (referred to as case-*noRC*) consisting of a three-story residential masonry building is selected. The geometry is inspired to an existing building located in San Felice sul Panaro (Italy) damaged by the seismic event that hit Emilia Romagna region in 2012 (Fig. 2a). It had exhibited a global seismic response with damage concentrated in the walls (mostly spandrels) without the activation of any out-of-plane mechanism



**Fig. 2** **a** Exterior view of the case study; **b** detail on one of the outer walls that highlights the activation of an in-plane response with cracks concentrated in masonry panels (mainly in spandrels)

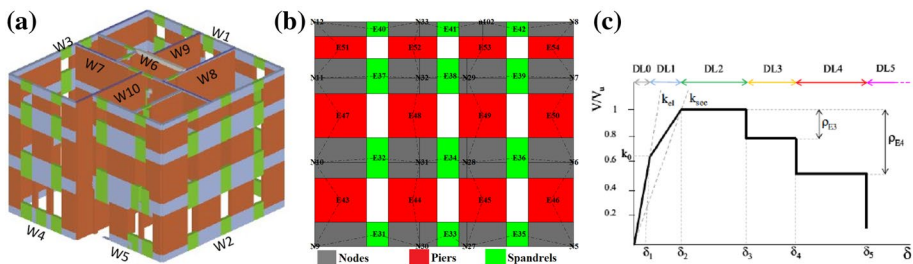
(Fig. 2b). The choice of such a building is motivated by the fact that the reliability of the modelling approach adopted has been already proven in previous studies (CNR-DT 212/2013 2014; Cattari and Lagomarsino 2013a) by simulating the actual seismic response occurred.

In particular, the three-story residential building is made of brick masonry and lime mortar. The diaphragms are made of concrete beams with flooring blocks, while the roof is a timber structure constructed with trusses and strut layers. The walls are characterized by a thickness of 24 cm and a brick masonry with lime mortar joints. The geometric and architectural configurations of the building are rather simple and regular, common in the area of examination as witnessed by other buildings in the vicinity.

The response of the structure is examined in the following through the equivalent frame modeling approach (Fig. 3a), using Tremuri program (Lagomarsino et al. 2013) and by performing nonlinear static analyses. The nonlinear response is concentrated into piers and spandrels (Fig. 3b) and described by nonlinear beam characterized by a piecewise linear constitutive law (Fig. 3c) that allows to describe the nonlinear monotonous response associated with increasing levels of damage (ending at collapse), by assigning progressive strength drops  $\rho_{Ei}$  at predetermined drift levels  $\delta_{Ei}$  (Cattari and Lagomarsino 2013b).

Starting from the same geometrical configuration and assuming the same materials mechanical properties, a second case study (referred to as case-RC) is analyzed by assuming at each floor the presence of reinforced concrete tie beams coupled with spandrels. This modification is motivated by the fact that the global response of masonry structures is quite sensitive to variations in structural details, such as the presence of tensile resisting elements coupled to spandrels. In this second case, it is expected that the structure tends to move from a failure mode with damage concentrated at the level of spandrels (case-noRC), to a soft story behavior (case-RC). This change in the global behavior is expected to correspondingly affect also the resulting sensitivity on the mechanical parameters.

Moreover, in this second case study, some parametric analyses were performed with different effective length of the RC tie beams, respectively equal to (Fig. 4b): the distance between two adjacent nodes; the width of the openings; or an intermediate length between the two. Although in reality the RC tie beams are obviously continuous at the floor level, these three alternatives aim to correspond to different hypotheses of the effectiveness of the actual restraint provided by the masonry on the RC elements. The results in terms of resulting pushover are presented in Fig. 4a in order to give an idea also of the potential effect of other sources of uncertainties inherent in the modelling that could be considered in the assessment; other examples are provided in CNR-DT 212/2013 (2014), Tondelli



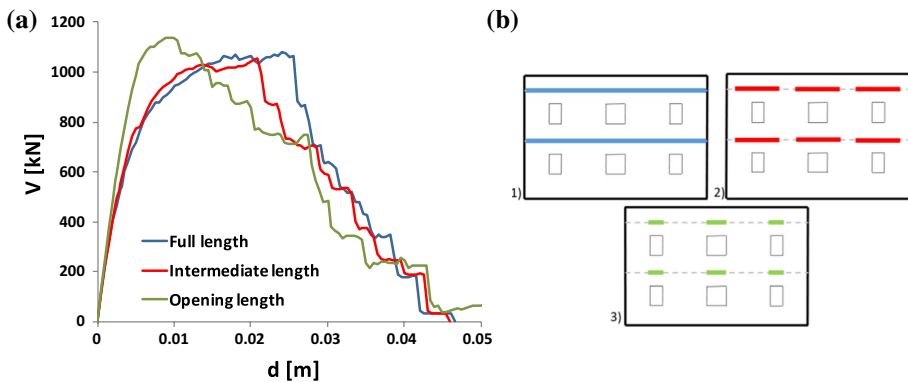
**Fig. 3** a 3D view of the structural model; b equivalent frame idealization of one of the outer walls; c Piecewise linear constitutive law adopted to simulate the nonlinear response of masonry panels

et al. (2012) and Cattari et al. (2015b) such as the presence of infilled openings, the quality of wall to wall and wall to diaphragms connections, the effectiveness of tie rods and that of lintels and architraves. On the following, by way of example, it was decided to deepen the case where the length is equal to the one of the opening (Fig. 4b-3).

### 4.2 Definition of the uncertainties described through aleatory variables

For both cases of study, ten aleatory variables (or group of variables)  $X_i$  are considered:

- $X_1$  *mechanical properties of masonry* It is a group of parameters comprising the modulus of elasticity  $E$ , the shear modulus  $G$ , the average shear strength  $f_{vm0}$ , the equivalent friction coefficient  $\mu$  and the compressive strength  $f_m$ . Further details on the strength criteria assumed to interpret the failure modes of panels are described in CNR-DT 212/2013 (2014). In particular, in the case of the shear failure mode, the adoption of these parameters reflects the choice of the criterion proposed by Mann and Müller (1980) for the interpretation of the diagonal cracking failure mode, which is considered the most representative for the brick masonry and lime mortar that characterizes the building.
- $X_2$  parameters that regulate the degradation of the initial elastic stiffness in the constitutive model assumed for masonry panels. It includes the parameters  $k_y$  and  $k_{in}$ . As indicated in Fig. 3c,  $k_y$  defines the value of the shear for which the stiffness degradation starts, while  $k_{in}$  is the ratio between the initial and the secant stiffness (see Cattari and Lagomarsino (2013b) for further explanations).
- $X_3$  *stiffness of the intermediate floors* It is described by the equivalent shear modulus  $G_{floor}$  assigned to the orthotropic membrane adopted in Tremuri program (Lagomarsino et al. 2013) to model the diaphragms (a conventional slab thickness equal to 4 cm is assumed).
- $X_4$  *stiffness of the roof* It is represented by the equivalent shear modulus  $G_{roof}$ .
- $X_5$  *stiffness of the stairs* It is represented by the equivalent shear modulus  $G_{stairs}$ .
- $X_6$  *response of masonry piers* It collects a group of parameters that mainly affect the softening phase of nonlinear response. In particular the group includes the drift ( $\theta_{M,T3}$ ,  $\theta_{M,T4}$ ,  $\theta_{M,T5}$ ,  $\theta_{M,PF3}$ ,  $\theta_{M,PF4}$ ,  $\theta_{M,PF5}$ ) and corresponding percentage of



**Fig. 4** Effect of modelling uncertainty associated to the effectiveness of RC tie beams coupled with spandrels (case-RC): **a** pushover curve; **b** 2D view of a wall with different effective length assumed for the coupled RC tie (1-full length; 2-intermediate length; 3-opening length)

residual strength ( $\rho_{M,T3}$ ,  $\rho_{M,T4}$ ,  $\rho_{M,PF4}$ ) associated to progressing damage levels. Both (drift and residual strength values) are differentiated as a function of two possible failure modes considered, where T refers to the diagonal cracking failure mode and PF to the rocking failure mode.

- $X_7$  *response of masonry spandrels* Analogously to piers, it collects a group of parameters that mainly affect the softening phase of nonlinear response in terms of drift ( $\theta_{F,T3}$ ,  $\theta_{F,T4}$ ,  $\theta_{F,T5}$ ,  $\theta_{F,PF3}$ ,  $\theta_{F,PF4}$ ,  $\theta_{F,PF5}$ ) and percentage of the residual strength ( $\rho_{F,T3}$ ,  $\rho_{F,T4}$ ,  $\rho_{F,PF4}$ ).
- $X_8$  *masses of intermediate floors* The sum of permanent and live loads (factored) is considered ( $p_{floor}$ ).
- $X_9$  *masses of the roof* The sum of permanent and live loads (factored) is considered ( $p_{roof}$ ).
- $X_{10}$  *masses of the stairs* The sum of permanent and live loads (factored) is considered ( $p_{stairs}$ ).

The ten uncertain parameters (or group of parameters) are considered completely independent, while within the same group the variables are considered completely correlated. It is worth noting that the adoption of the star design with central point approach is consistent only under the hypothesis of independent variables, while it can provide only approximate results when it isn't satisfied. Indeed, the proposed method aims to be practice oriented and useful also to support the setting of the investigation plan, this is why the choice of executing only  $2N + 1$  analyses, together with the possibility to compute the partial dispersion, results particularly effective. In the specific case of masonry structures, that are the subject of the application in this study, statistical data from experimental tests inherent the mechanical parameters are not enough to establish robust correlation laws; thus, the assumption of considering the parameters belonging to the first group ( $X_1$ ) as fully correlated appears licit, although to some extent approximate. In other applications (Haddad et al. 2019), the compressive strength and the parameters associated to the shear strength have been considered as independent: that represents a possible alternative—equally licit and conventional for the reason aforementioned—particularly useful to address the choice in the most reliable investigation technique to be adopted (e.g. if the double flat jacket test, in the case of masonry panels dominated by a flexural response, or other tests—like as the shove test or the diagonal compressive test—more appropriate to investigate the shear parameters when masonry panels are mainly affected by a diagonal shear cracking).

Table 1 summarizes the plausible ranges of variation assumed for each parameter in two cases representative of the basic and improved knowledge levels acquired before and after the execution of the investigation phase, respectively; this two cases are referred in the following as path 1 and 2.

Variables  $X_3$ ,  $X_4$  and  $X_5$  are considered as uncertainties since they have a great impact on the redistribution of the forces among the linear and non-linear walls. In this case, the significant variation associated to the floor, roof and stair stiffness reflects not only the uncertainty of mechanical properties but also the quality of connection with the perimeter walls that could compromise the actual capacity of the diaphragm to transfer the seismic actions despite its own stiffness. The uncertainties of loads on diaphragms ( $X_8$ ,  $X_9$ , and  $X_{10}$ ) reflect those on the finishing and, for example, the thickness of the slab, in case of intermediate floors and stairs. The values of these uncertain parameters are defined starting from those corresponding to normal weight concrete, for intermediate floors and stairs, and to timber, for the truss roof.



**Table 1** Plausible ranges of variation for all the uncertain parameters assumed in both cases of study, represented by the lower, upper and median values; dispersions assumed for lognormal distributions ( $\Delta_{\log}$ ) and a and b parameters assumed for the beta distributions used in the Monte Carlo simulation before (in black) and after the investigation phase (in brackets and in italic)

Aleatory variables	$X_{low}$	$X_{up}$	$X_{med}$	$\Delta_{\log}$	a	b
$X_1$						
$E$ [MPa]	600(780)	1350(1170)	900	0.41(0.2)	–	
$G$ [MPa]	200(260)	450(390)	300	0.41(0.2)	–	
$f_{mvo}$ [MPa]	0.1(0.128)	0.1875(0.16)	0.137	0.26(0.06)	–	
$\mu$	0.333(0.421)	0.5625(0.474)	0.433	0.31(0.11)	–	
$f_m$ [MPa]	2.4(3.15)	6(5.25)	3.795	0.46(0.26)	–	
$X_2$						
$k_y$	0.5(0.585)	0.8(0.715)	0.65	–	1.5(2.625)	1.5(2.625)
$k_{in}$	1.25(1.29)	1.751.71	1.5	–	5.922(34.35)	3.189(18.5)
$X_3$						
$G_{floor,eq}$ [MPa]	1250(3904)	12500(9795)	3953	1.15(0.46)	–	
$X_4$						
$G_{roof,eq}$ [MPa]	100(312)	1000(784)	316	1.15(0.46)	–	
$X_5$						
$G_{stairs,eq}$ [MPa]	1250(3904)	12500(9795)	3953	1.15(0.46)	–	
$X_6$						
$\theta_{M,T3}$	0.00229	0.00371	0.00291	0.24	–	
$\theta_{M,T4}$	0.00392	0.00608	0.00488	0.22	–	
$\theta_{M,T5}$	0.00562	0.00838	0.00686	0.2	–	
$\theta_{M,PF3}$	0.00459	0.00741	0.00583	0.24	–	
$\theta_{M,PF4}$	0.00783	0.01216	0.00976	0.22	–	
$\theta_{M,PF5}$	0.01204	0.01796	0.0147	0.2	–	
$\rho_{M,T3}$	0.6	0.8	0.7	–	14	6
$\rho_{M,T4}$	0.25	0.55	0.4	–	3.867	5.8
$\rho_{M,PF4}$	0.8	0.9	0.85	–	42.5	7.5
$X_7$						
$\theta_{F,T3}$	0.00153	0.00247	0.00194	0.24	–	
$\theta_{F,T4}$	0.00453	0.00747	0.00582	0.25	–	
$\theta_{F,T5}$	0.0151	0.02489	0.0194	0.25	–	
$\theta_{F,PF3}$	0.00153	0.00247	0.00194	0.24	–	
$\theta_{F,PF4}$	0.00453	0.00747	0.00582	0.25	–	
$\theta_{F,PF5}$	0.0151	0.02489	0.0194	0.25	–	
$\rho_{F,T3}$	0.3	0.7	0.5	–	2.625	2.625
$\rho_{F,T4}$	0.3	0.7	0.5	–	2.625	2.625
$\rho_{F,PF4}$	0.3	0.7	0.5	–	2.625	2.625
$X_8$						
$P_{floor}$ [kN/m <sup>2</sup> ]	0.805(0.94)	1.196(1.06)	0.981	0.2(0.06)	–	
$X_9$						
$P_{roof}$ [kN/m <sup>2</sup> ]	0.8(0.94)	1.2(1.06)	0.98	0.2(0.06)	–	
$X_{10}$						
$P_{stairs}$ [kN/m <sup>2</sup> ]	0.805(0.94)	1.196(1.06)	0.981	0.2(0.06)	–	

The values of the mechanical parameters of masonry are defined from the proposed values in MIT (2009), commentary of the NTC (2008). Such document proposes reference values for various masonry typologies characterizing existing buildings; moreover, together with values representative of a state considered not conforming to all the rules-of-art, specific corrective factors are proposed in order to account for the positive effect of specific structural details like as the presence of a good mortar quality, the good transversal connection between leaves, etc. The range of variation proposed in Table 1 is defined starting from the basic reference values and then considering—for defining the upper value—the application of the correction factor proposed in MIT (2009) to include the possible influence of a good mortar quality (equal to 1.5). The ranges of variation of the parameters that regulate the stiffness degradation and the drift limit of the piers and spandrels are calibrated using the data available from reference literature (Morandi et al. 2018), or from some experimental campaigns (Anthoine et al. 1995; Petry and Beyer 2014) and by referring simultaneously to the analysis performed in CNR-DT 212/2013 (2014) on this specific structure.

### 4.3 Execution of the model parameter sensitivity analysis

For the execution of the sensitivity analysis, nonlinear static analyses are performed in X and Y directions, in the two senses, positive and negative, for both case studies, with load pattern distributed proportionally to masses. The latter choice derives from the evidences collected from previous numerical simulations performed on this structure (CNR-DT 212/2013 2014), through the execution of nonlinear dynamic analyses that proved that this load pattern is the most reliable in simulating the actual seismic response of this structure.

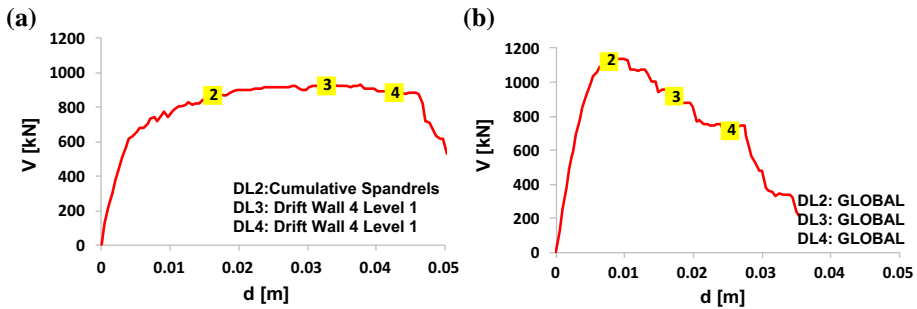
Results achieved in X and Y directions are herein analyzed separately considering them as derived from two different structures, being the main aim to assess the effectiveness of the procedure proposed more than the strict verification of the building.

Usually, in PBA four Limit States (LSs) related to serviceability and ultimate conditions of the structure (EC8-part 3 EN1998-3 2005; ASCE/SEI 41-13 2014) are considered. For the case studies analyzed, reference is made to the attainment of progressing damage levels (DLs) assumed to correspond to the Damage Limitation, Life Safety, and Collapse limit states as provided in EC8-part 3 (EN1998-3 2005), from here on, more generally, named as DL2, DL3 and DL4, respectively. The position of the DLs on the pushover curve is defined using the multiscale approach proposed in Lagomarsino and Cattari (2015a, b), that combines checks at three different scales: structural element scale, macroelement (walls) scale, and global scale. At each scale specific variables are introduced, that are: at element scale, the cumulative damage of the elements (piers or spandrels) that have reached a predetermined DL (as corresponding to the attainment of given drift thresholds as shown in Fig. 3c and specified in Table 1); at macroelement scale, the inter-story drift; and at global scale, the maximum base shear as defined by the pushover curve. The attainment of the DL for each scale is checked by introducing proper thresholds; those herein assumed are summarized in Table 2. The final position of the DL on the pushover curve corresponds to the worst among the three scales.

The value of the PGA is calculated using the Capacity Spectrum Method (Freeman 1998) based on the overdamped spectrum approach. For the conversion of the pushover into the equivalent single degree of freedom, reference is made to the principles proposed in EC8-part 3 (EN1998-3 2005) and NTC (2008), by computing the equivalent mass  $m^* = \sum m_i \Phi_i$  and the participation factor  $\Gamma = m^* / \sum m_i \Phi_i^2$  as originally proposed in Fajfar

**Table 2** Limit thresholds assumed to define the DLs at the macroelement and global scales, where  $\theta_i$  refers to the interstory drift and  $\rho_{Gi}$  refers to the strength reduction on the pushover curve

DLi	Macroelement scale	Global scale
DL2	$\theta_2=0.3$	–
DL3	$\theta_3=0.5$	$\rho_{G3}=0.2$
DL4	$\theta_4=0.7$	$\rho_{G4}=0.4$



**Fig. 5** Positions of DLs on pushover curves in the X direction for **a** case-noRC and **b** case-RC

(2000). Concerning the seismic demand, it was conventionally decided to adopt the one already used in the application illustrated CNR-DT 212/2013 (2014), that derives from the median value of response spectra of 30 recordings selected from disaggregated data selected in a range of magnitude between the values 5.6 and 6.5, and of distance between 10 and 30 km; the database used consists in an aggregation of European ESD databases and the Italian databases SIMBAD (Smerzini et al. 2014) and ITACA (<http://itaca.mi.ingv.it/>).

Figure 5a, b show a comparison between the pushover curves obtained for the two models (case-noRC and case-RC, respectively) in the X direction, that highlights the modification in the building global response already mentioned in Sect. 4.1. In particular, the structure moves from a ductile but less resistant behavior in case-noRC to a more brittle and stronger behavior in case-RC; the scales dominant in defining the position of the three DLs consist in the global one in case-RC, while in the element (particularly the spandrels) and macroelement scales in case-noRC.

As mentioned in Sect. 3, the complete way to execute a model parameter sensitivity analysis should be through a full factorial one exploring all the possible combinations among the different uncertain parameters (or set of parameters). Here the latter is executed to investigate if the star design with central point approach is capable or not to capture accurately the parameters that mostly affect the seismic response of the structure. For both cases of study,  $2^N=1024$  and  $2N+1=21$  nonlinear static analyses have been thus performed. The collected  $im_{LS}$  are then used in Eq. (4) to generate the  $\beta x$  values. Figure 6 shows the comparison of the obtained results.

Assuming the complete factorial analysis as reference, it results that in most of cases the  $2N+1$  analyses are capable to capture the parameters with highest sensitivities among the ten aleatory variables considered. Further details on how the results in terms of partial dispersion can be adopted for the aim of supporting the setting of the investigation are illustrated in Haddad et al. (2017). In the following, it will be assumed that the execution of a proper investigation plan allowed to increase the knowledge on some parameters ( $X_1, X_2,$

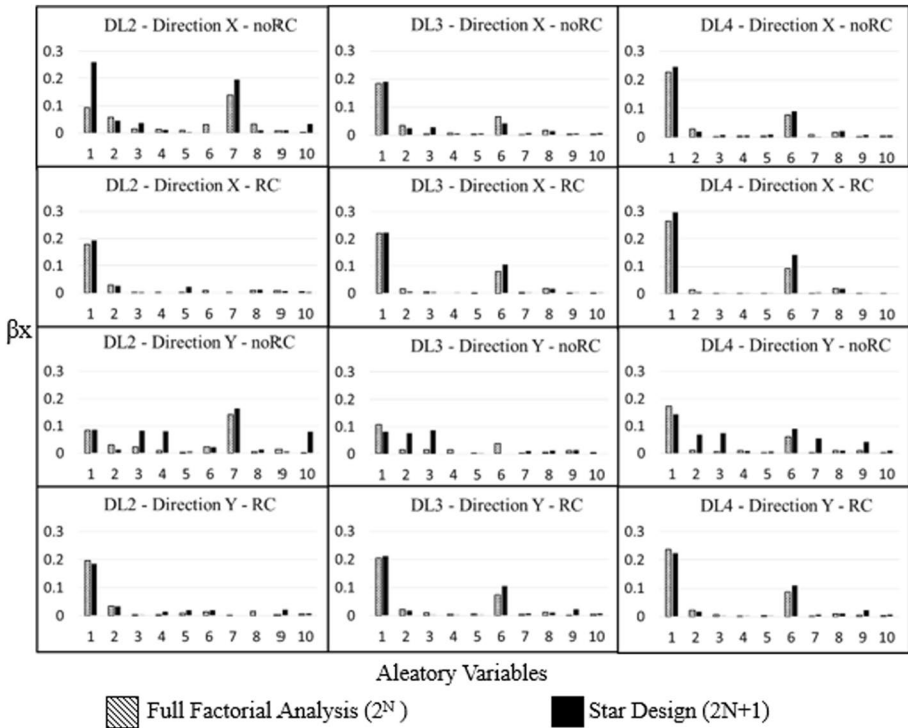


Fig. 6 Partial Dispersions  $\beta_x$  obtained for the 10 aleatory variables considered

$X_3, X_4, X_5, X_8, X_9, X_{10}$ ) and thus to define an updated set of values useful for proceeding to the final safety assessment, as illustrated in Sect. 4.4.

#### 4.4 Generation of the reference $IM_{LS}$ and $\beta_{LS}$ values using Monte Carlo sampling

The approach used for the generation of the median values for  $IM_{LS}$  and  $\beta_{LS}$  to be adopted as reference for the validation of the proposed procedure is the evaluation of fragility curves through the application of a fully probabilistic procedure based on the Monte Carlo sampling.

The approach requires the attribution of an appropriate probability density function to each aleatory variable and thus the definition of the parameters that characterize it. In other words, the minimum and maximum values of the plausible range of variation defined for the execution of the star design with central point approach need here to assume a statistical meaning in terms of fractiles. In particular, it is assumed that the lower and upper values previously assumed correspond to the 16% and 84% fractiles, respectively.

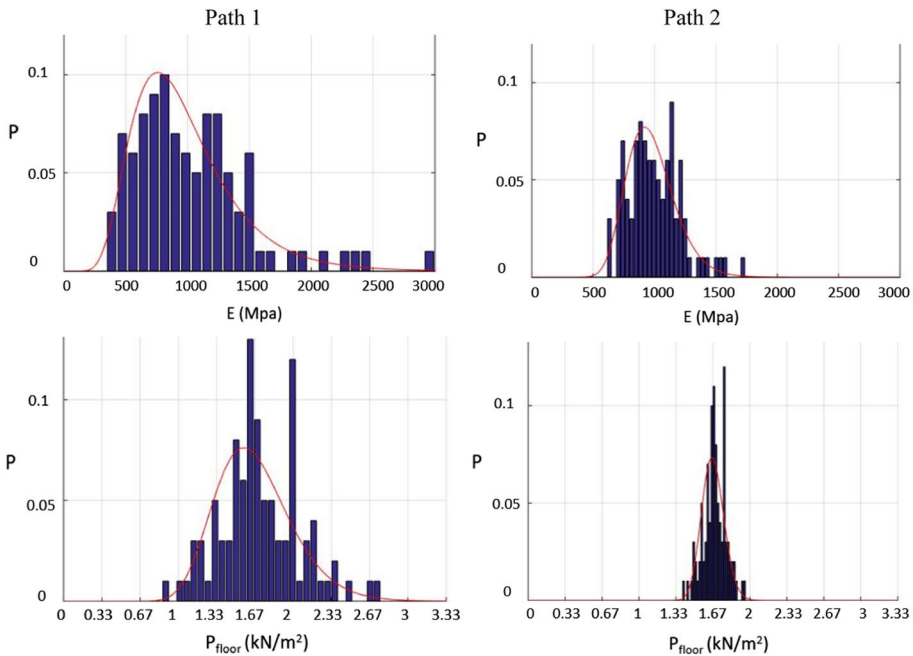
Other sampling techniques than the simple Monte Carlo one could have been used as well, such as the Latin Hypercube sampling (e.g. used by Dolsek 2009; Fragiadakis and Vamvatsikos 2010), that is more efficient and less time consuming, requiring in general a lower number of analyses. The choice of this simplest not optimized sampling method is justified by the fact that the construction of fragility curves based on a fully

probabilistic approach constitutes only a validation tool and isn't a required step of the proposed procedure.

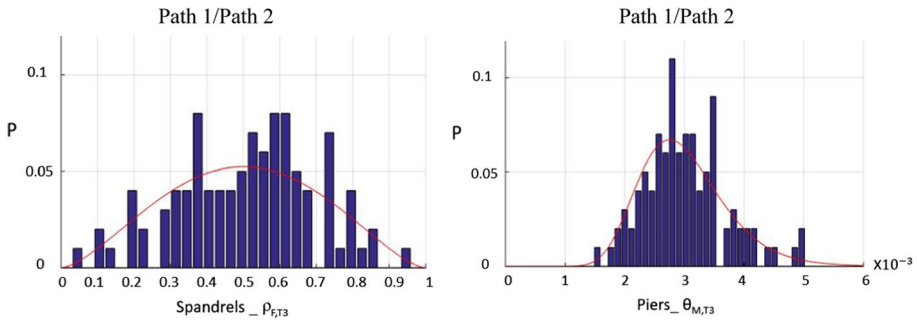
Lognormal distributions are assumed for the parameters having values ranging between 0 and infinity ( $X_1, X_3, X_4, X_5, X_8, X_9, X_{10}$ ) while beta distribution for those varying between 0 and 1 ( $X_6, X_7$ ) or having, from a physical point of view, a limited range of variation ( $X_2$ ). Examples of the two kinds of distributions (lognormal and beta) and their sampling, for the two knowledge paths, are illustrated in Figs. 7 and 8.

For each variable, the parameters assumed for the corresponding probability density functions are differentiated for the two knowledge levels simulated (path 1 and 2, respectively) (see Table 1). In particular, for all groups of parameters for which it has been assumed reasonable improvement of the knowledge, standard deviations are assumed higher in path 1 (before the execution of the investigation plan) than in path 2. As evident from Table 1, only the dispersions associated to groups 6 and 7 are maintained invariant being associated to the description of the softening phase of piers and spandrels for which a significant and reliable improvement in the knowledge would require very invasive in situ tests (thus usually not feasible).

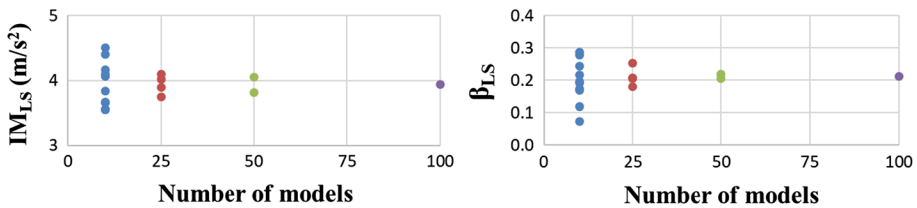
The generation of a number of models is done using Monte Carlo technique. In particular, 100 models for each case study are generated verifying that such a number is sufficient from a statistical point of view. To this aim, it has been verified that the selected number of samples is sufficient to reach a good convergence in the estimation of two the parameters that define the fragility curves. Figure 9 illustrates the result of



**Fig. 7** Examples of lognormal distributions assumed for the aleatory variables for which it has been assumed possible an improvement of the knowledge passing from path 1 to path 2. Histograms refers to the sampling of 100 values made through the Monte Carlo technique



**Fig. 8** Example of beta (for the residual strength of spandrels,  $\rho_{FT3}$ ) and lognormal (for the drift limits of piers,  $\theta_{M,T3}$ ) distributions assumed for the aleatory variables for which it has been assumed impossible an improvement of the knowledge passing from path 1 to path 2. Histograms refers to the sampling of 100 values made through the Monte Carlo technique



**Fig. 9** Comparison of the  $IM_{LS}$  and  $\beta_{LS}$  values obtained from the simulation of 10–100 models for the case-noRC in the X direction (DL4)

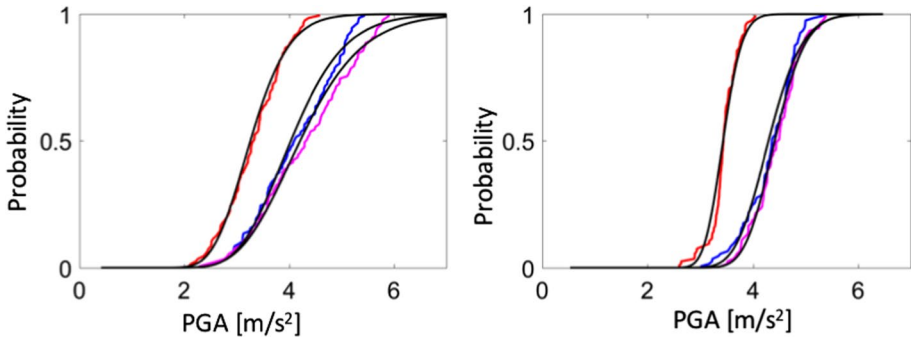
such convergence check by showing the parameters  $IM_{LS}$  and  $\beta_{LS}$  achieved considering progressively 10, 25, 50 and finally 100 models.

The execution of the nonlinear static analyses allows the construction of the fragility curves differentiated for each direction and each DL. They are obtained by taking the lower PGA between the two directions (positive and negative) and putting in an ascending order the values obtained for the three DLs considered. Out of these numerical fragilities, it is calculated a median value  $IM_{LS}$  and a dispersion  $\beta_{LS}$ , used to fit these fragility curves by lognormal ones.

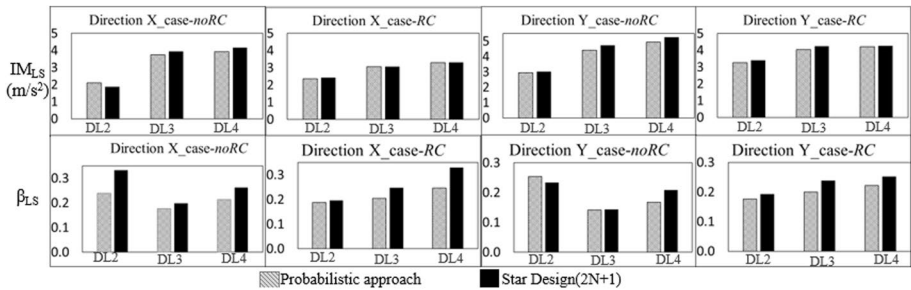
An example of the numerical curves and their lognormal fit is illustrated in Fig. 10 for the two knowledge paths. It is noticeable the change in the slope and the dispersion of the fragility curves moving from the first to the second path, where the distributions of the uncertain parameters are lower.

Figures 11 and 12 summarize the results obtained for the cases of study examined in the two paths of knowledge, respectively. From these figures, it is clear how the adoption of the star design with central point approach is able to provide, in almost all cases, a median value of  $IM_{LS}$  close to the reference one generated by the probabilistic approach. On the other hand, the comparison of the  $\beta_{LS}$  values between the two approaches shows again a reasonable compatibility with a median difference reaching around 3%, for both knowledge levels. Indeed, the differences are considerably acceptable and occur only in very few cases. The afore mentioned results lead to the preliminary conclusion that relying on a limited number of analyses (in this case,  $2N + 1$  analyses) allows to correctly estimate

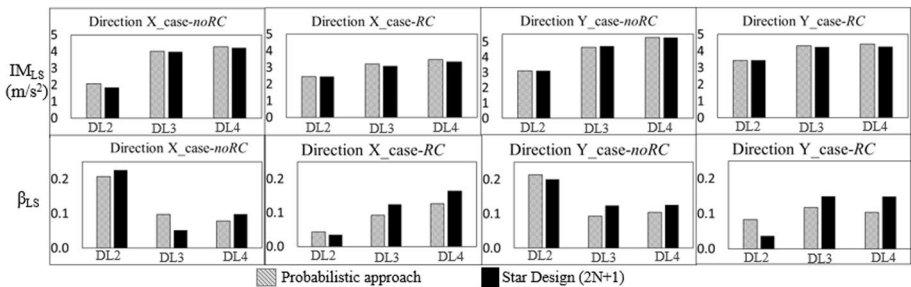




**Fig. 10** Numerical fragility curves and their lognormal fit for the three DLs in the case of nonlinear static analyses performed on case-RC in Y direction: **a** path 1 (basic knowledge); **b** path 2 (after investigation)



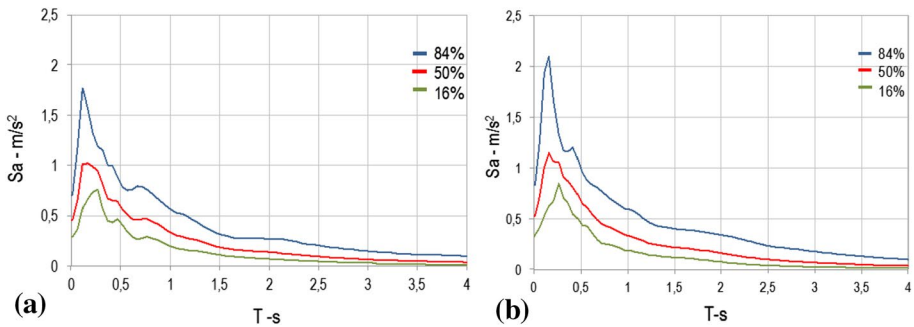
**Fig. 11** Comparison of the  $IM_{LS}$  and  $\beta_{LS}$  values of the fragility curves obtained from the probabilistic approach and the star design with central point approach for the 1st path of knowledge



**Fig. 12** Comparison of the  $IM_{LS}$  and  $\beta_{LS}$  values of the fragility curves obtained from the probabilistic approach and the star design with central point approach for the 2nd path of knowledge

the fragility curve representing the capacity of the structure under examination. This is true under the hypothesis that it is reasonable to assume the various groups of variables as uncorrelated.

Since, as introduced in Sect. 4.3, the median response spectrum adopted to compute the  $IM_{LS}$  values derives from the post-processing of 30 recordings properly selected, in this case it is easy computing also the 16% and 84% fractiles, as shown in Fig. 13 where they have been computed considering the geometrical mean of two components (NS and WE)



**Fig. 13** Median spectrum and those corresponding to the fractiles 16% and 84% used in: **a** X direction; **b** Y direction (from CNR DT 212 2013)

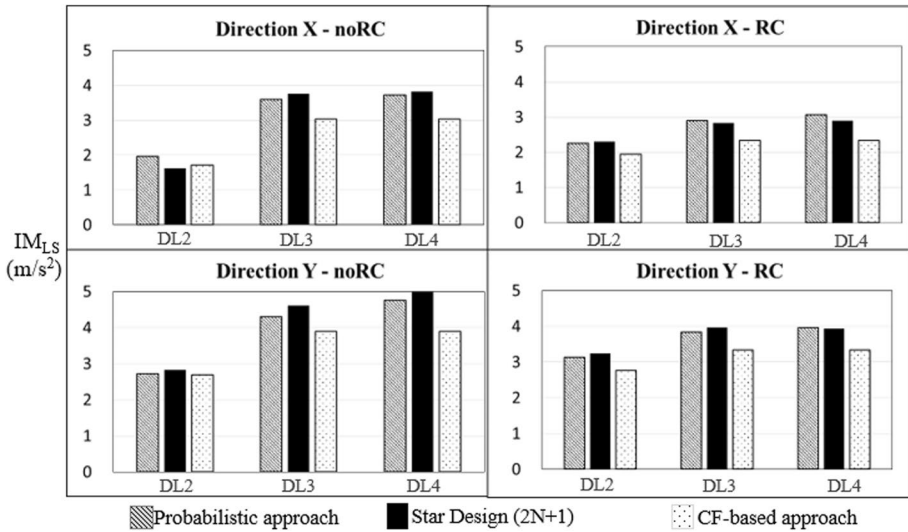
available. With these data, also the contribution of  $\beta_{LS,D}$  has been computed in the case of the knowledge path 1 resulting in values in average ranging from 0.31 to 0.44 passing from DL2 to DL4. It results that the contribution associated to the record-to-record variability is even higher than that coming from the model parameter uncertainty, as already testified in many other applications in literature, e.g. recently within the RINTC Workgroup 2018 as a function of various structural typologies, like as reinforced concrete, masonry, steel and precast (Iervolino et al. 2018; Cattari et al. 2018). This highlights the potential strong approximation currently adopted by codes in the format of the seismic safety assessment.

#### 4.5 Probabilistic-based and CF-based safety assessment

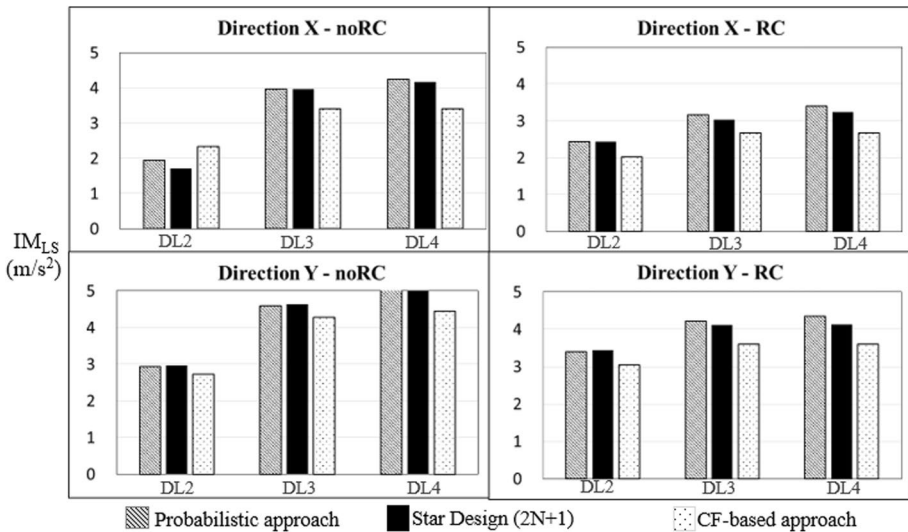
In the following the code based procedure based on the use of the CF (Sect. 2) is applied to the two case studies together with the probabilistic-based procedure proposed in Sect. 3.4.

In particular, the CF-based assessment is carried out according to the recommendations of EC8-part 3 (EN1998-3 2005) applying the CF factor to the mechanical properties of masonry (group of parameters collected in  $X_1$ ). Coherently with the two paths afore introduced, two knowledge levels are assumed in which the model adopted for the safety assessment is characterized by the median values for all the uncertain variables apart those associated to the set  $X_1$  in which they are divided respectively: by 1.35, in the state representative of that before the investigation phase (path 1); and by 1.2, in the state of knowledge after investigation (path 2). Such two values are those proposed in EC8-part 3 (EN1998-3 2005) in the case of attainment of the knowledge levels KL1 and KL2, respectively.

In Figs. 14 and 15 the comparison of the IM values obtained from the three different approaches adopted is represented, that are: the full probabilistic approach, the probabilistic-based approach proposed in the paper that exploits results of the star design with central point analyses, and the CF-based approach. In the first case the IM value corresponds to that associated to a probability equal to 0.5 in the fragility curve, in the second case it corresponds to the  $IM^*$  value (as introduced in Sect. 3.4), while in the latter it corresponds to the value obtained from the execution of the nonlinear static analysis in which median values of strength parameters are divided by the CF. For the computation of  $IM^*$ , according to the seismic input assumed as introduced in Sect. 4.3, the values of  $k_0$  and  $k$  are assumed 0.05 and 2.5, respectively.



**Fig. 14** Comparison of the IM values obtained in the 1st path of knowledge from the three different approaches: full probabilistic, star design with central point and CF-based



**Fig. 15** Comparison of the IM values obtained in the 2nd path of knowledge from the three different approaches: probabilistic, model parameter sensitivity analysis and CF-based

The compatibility between the values of IM obtained from the full probabilistic approach, based on the result of Monte Carlo simulation, and from the results of the model parameter sensitivity analysis confirms the preliminary conclusion set in Sect. 4.4 that the execution of the limited number of analyses equal to  $(2N + 1)$  is sufficient to safely estimate the capacity of the assessed structure, even if, in some cases, the

adopted total dispersion  $\beta_{LS}$  slightly differs from the one obtained from the probabilistic approach.

On the other hand, the analyses performed according to the CF-based procedure produce values of IM that are, in almost all cases, lower than those obtained from the probabilistic approach or the star design with central point approach. Result achieved in these cases of study show that the CF-based procedure may be over-conservative, reaching in some cases a difference of 25% with the reference one. Indeed, it is consistent with the fact that the CF was applied to the mechanical properties of masonry that in the two case studies presented in this paper showed the highest degree of uncertainty. However, this cannot be considered a general result as already proven by other authors (Tondelli et al. 2012).

In fact, a different result is expected if similar levels of sensitivity are associated to different parameters or if there is any parameter having a sensitivity higher than the mechanical properties of masonry. It is important also to observe that changing in the model the value only of a single parameter may lead to uncontrolled change in the whole seismic behavior and in the global collapse mechanism, especially if this parameter is related to the mechanical properties of masonry that play an important role in the structural response. For example, the reduction only of the shear strength could lead to pass from a prevailing flexural behavior to a prevailing diagonal cracking failure mode with a significant change in the global ductility (since the drift limits associated to two failure modes are significantly different). Moreover, in some cases the difference obtained between the CF-based IM values and the reference ones is quite high highlighting one of the main drawbacks of the CF-based approach (already mentioned in Sect. 2) that is the logic of adopting pre-defined values of CF indiscriminately valid for all structures. The latter issue is solved by the proposed procedure that, thanks to the execution of the model parameter sensitivity analysis, is able to compute a value of the confidence factor CF\* targeted to the specific characteristics of the structure and the incomplete residual knowledge.

## 5 Conclusions

Various studies available in literature proved the inadequacy of the CF-based approach, which is currently adopted in all international standards and guidelines as standard approach for including the residual incomplete knowledge in the seismic assessment of existing buildings.

Although at research level or in the case of very relevant applications, the full probabilistic approach appears as the more appropriate alternative to be pursued, it is usually based on the execution of complex and high number of analyses. Being such approach highly demanding and hard to be proposed as a practice-oriented procedure for engineers, at least nowadays, it rises the need for effective alternatives to the conventional approach proposed by codes.

Within this context and recognizing the highest versatility and accuracy of passing to a full probabilistic approach, in this paper the potential of the use of a limited number of analyses is explored in order to compute the two basic parameters ( $IM_{LS}$  and  $\beta_{LS}$ ) necessary to compute the fragility curve and the probability of occurrence of a given limit state. In particular, a model parameter sensitivity analysis is performed according to the star design with central point approach, that requires the execution only of  $2N + 1$  analyses. This approach is very convenient to limit the number of analyses, even it provides consistent results only when it is reasonable to assume the uncertain parameters (or set

of parameters) involved in the assessment as independent. Despite such a limitation, the sensitivity analysis is proposed in that way since results in terms of partial dispersions are very effective also to support the design of the investigation plan, aiming to decrease the cost and invasiveness of the tests performed but at the same time increase the knowledge on parameters mostly affecting the structural response.

As introduced in Sect. 3.4, the computation of the  $IM_{LS}$  and  $\beta_{LS}$  parameters could be converted in a convenient format also for practice-oriented procedures as already proposed in a similar way in Cornell et al. (2002).

In the paper, a first application on two URM case studies is presented. The comparison with values obtained from fragility curves built through the execution of a large number of nonlinear static analyses on models generated using Monte Carlo simulations, proved that the use of such targeted but limited number of analyses is quite effective and is able to guarantee results on the safe side.

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## References

- Anthoine A, Magonette G, Magenes G (1995) Shear-compression testing and analysis of brick masonry walls. In: Duma (ed) Proceedings 10th European conference on earthquake engineering, Wien, Austria. Balkema, Rotterdam
- ASCE/SEI 41-13 (2014) Seismic evaluation and retrofit of existing buildings. American Society of Civil Engineers, Reston. ISBN 978-0-7844-7791-5
- Bisch P (2018) Eurocode 8. Evolution or revolution? In: Ptilakis K (eds) Recent advances in earthquake engineering in Europe. ECEE 2018. Geotechnical, geological and earthquake engineering, vol 46. Springer, Cham
- Bracchi S, Rota M, Magenes G, Penna A (2016) Seismic assessment of masonry buildings accounting for limited knowledge on materials by Bayesian updating. Bull Earthq Eng 14(8):2273–2297. <https://doi.org/10.1007/s10518-016-9905-8>
- Cattari S, Lagomarsino S (2013a) Seismic assessment of mixed masonry-reinforced concrete buildings by non-linear static analyses. Earthq Struct 4(3):241–264
- Cattari S, Lagomarsino S (2013b) Masonry structures. In: Sullivan T, Calvi GM (eds) Developments in the field of displacement based seismic assessment. IUSS Press (PAVIA) and EUCENTRE, pp 151–200. ISBN: 978-88-6198-090-7
- Cattari S, Lagomarsino S, Bosiljkov V, D'Ayala D (2015a) Sensitivity analysis for setting up the investigation protocol and defining proper confidence factors for masonry buildings. Bull Earthq Eng 13(1):129–151. <https://doi.org/10.1007/s10518-014-9648-3>
- Cattari S, Lagomarsino S, Karatzetou A, Ptilakis D (2015b) Vulnerability assessment of Hassan Bey's Mansion in Rhodes. Bull Earthq Eng 13(1):347–368. <https://doi.org/10.1007/s10518-014-9613-1>
- Cattari S, Camilletti D, Lagomarsino S, Bracchi S, Rota M, Penna A (2018) Masonry Italian code-conforming buildings. Part 2: nonlinear modelling and time-history analysis. J Earthq Eng. <https://doi.org/10.1080/13632469.2018.1541030>
- CNR-DT 212/2013 (2014) Guide for the probabilistic assessment of the seismic safety of existing buildings. National Research Council, Rome
- Cornell CA, Jalayer F, Hamburger RO, Foutch DA (2002) Probabilistic basis for 2000 SAC federal emergency management agency steel moment frame guidelines. J Struct Eng 128(4):526–533
- Dolsek M (2009) Incremental dynamic analysis with consideration of modeling uncertainties. Earthq Eng Struct Dyn 38(6):805–825. <https://doi.org/10.1002/eqe.869>
- EN1998-3 (2005) Eurocode 8-design of structures for earthquake resistance. Part 3: assessment and retrofitting of buildings. Brussels, Belgium

- Fajfar P (2000) A non linear analysis method for performance-based seismic design. *Earthq Spectra* 16(3):573–591
- Fragiadakis M, Vamvatsikos D (2010) Incremental dynamic analysis for estimating seismic performance sensitivity and uncertainty. *Earthq Eng Struct Dyn* 39(2):141–163. <https://doi.org/10.1002/eqe.935>
- Franchin P, Pagnoni T (2018): A general model of resistance partial factors for seismic assessment and retrofit. In: Proceedings of the 16th European conference on earthquake engineering, Thessaloniki, Greece
- Franchin P, Pinto PE, Pathmanathan R (2010) Confidence factor? *J Earthquake Eng* 14:989–1007
- Franchin P, Ragni L, Rota M, Zona A (2018) Modelling uncertainties of Italian code-conforming structures for the purpose of seismic response analysis. *J Earthq Eng*. <https://doi.org/10.1080/13632469.2018.1527262>
- Freeman SA (1998) Development and use of capacity spectrum method. In: Proceedings of the 6th US national conference of earthquake engineering, Seattle, USA
- Haddad J, Cattari S, Lagomarsino S (2017) Preliminary and sensitivity analysis to set the investigation plan for the seismic assessment of existing buildings. In: Proceedings of the 3rd international conference on protection of historical constructions, Lisbon, Portugal
- Haddad J, Cattari S, Lagomarsino S (2019) Sensitivity and preliminary analysis for the seismic assessment of Ardinghelli Palace. In: Proceedings of the 11th international conference on structural assessment of historical constructions Cuzco, Perú. RILEM Bookseries, vol 18, pp 2412–2421
- Iervolino I, Spillatura A, Bazzurro P (2018) Seismic structural reliability of code-conforming Italian buildings. *J Earthq Eng*. <https://doi.org/10.1080/13632469.2018.1540372>
- Jalayer F, Cornell CA (2003) A technical framework for probability-based demand and capacity factor design (DCFD) seismic format, PEER report. Pacific Earthquake Engineering Center, College of Engineering, University of California Berkeley
- Jalayer F, Elefante L, Iervolino I, Manfredi G (2011) Knowledge-based performance assessment of existing RC buildings. *J Earthq Eng* 15:362–389
- Lagomarsino S, Cattari S (2015a) PERPETUATE guidelines for seismic performance-based assessment of cultural heritage masonry structures. *Bull Earthq Eng* 13(1):13–47. <https://doi.org/10.1007/s10518-014-9674-1>
- Lagomarsino S, Cattari S (2015b) Seismic performance of historical masonry structures through pushover and nonlinear dynamic analyses. *Geotech Geol Earthq Eng* 39:265–292. [https://doi.org/10.1007/978-3-319-16964-4\\_11](https://doi.org/10.1007/978-3-319-16964-4_11)
- Lagomarsino S, Penna A, Galasco A, Cattari S (2013) TREMURI program: an equivalent frame model for the nonlinear seismic analysis of masonry buildings. *Eng Struct* 56:1787–1799
- Liel AB, Haselton CB, Deierlein GG, Baker JW (2009) Incorporating modeling uncertainties in the assessment of seismic collapse risk of buildings. *Struct Saf* 31(2):197–211
- Mann W, Müller H (1980) Failure of shear-stressed masonry—an enlarged theory, tests and application to shear-walls. In: Proceedings of international symposium on load-bearing brickwork, London, UK, pp 1–13
- Marino S, Cattari S, Lagomarsino S (2018) Use of non-linear static procedures for irregular URM buildings in literature and codes. In: Proceedings of the 16th European conference on earthquake engineering, June 18–21, Thessaloniki, Greece
- MIT (2009) Ministry of Infrastructures and Transportation, Circ. C.S.LI. pp. No. 617 2/2/2009, Istruzioni per l'applicazione delle nuove norme tecniche per le costruzioni di cui al Decreto Ministeriale 14 Gennaio 2008, G.U. S.O. n.27 of 26/2/2009, No. 47 (in Italian)
- Morandi P, Albanesi L, Graziotti F, Li Piani T, Penna A, Magenes G (2018) Development of a dataset on the in-plane experimental response of URM piers with bricks and blocks. *Constr Build Mater*. <https://doi.org/10.1016/j.conbuildmat.2018.09.070>
- NTC (2008) Ministerial decree 14/1/2008. Technical standards for buildings. Italian Ministry of Infrastructure and Transportation. G.U. S.O. n.30 of 4/2/2008 (in Italian)
- Petry S, Beyer K (2014) Cyclic test data of six unreinforced masonry walls with different boundary conditions. *Earthq Spectra*. <https://doi.org/10.1193/101513eqs269>
- Pinto PE, Giannini R, Franchin P (2004) Seismic reliability analysis of structures. IUSS Press, Pavia. ISBN 88-7358-017-3
- RINTC Workgroup (2018) Results of the 2015–2017 implicit seismic risk of code-conforming structures in Italy (RINTC) project. ReLUIS-EUCENTRE report. <http://www.reluis.it/>
- Rota M, Penna A, Magenes G (2014) A framework for the seismic assessment of existing masonry buildings accounting for different sources of uncertainty. *Earthq Eng Struct Dyn* 43(7):1045–1066. <https://doi.org/10.1002/eqe.2386>



- Smerzini C, Galasso C, Iervolino I, Paolucci P (2014) Ground motion record selection based on broadband spectral compatibility. *Earthq Spectra* 30(4):1427–1448
- Tondelli M, Rota M, Penna A, Magenes G (2012) Evaluation of uncertainties in the seismic assessment of existing masonry buildings. *J Earthq Eng* 16(S1):36–64
- Vamvatsikos D (2013) Derivation of new SAC/FEMA performance evaluation solutions with second-order hazard approximation. *Earthq Eng Struct Dyn* 42(8):1171–1188. <https://doi.org/10.1002/eqe.2265>
- Vamvatsikos D (2014) Accurate application and second-order improvement of SAC/FEMA probabilistic formats for seismic performance assessment. *ASCE J Struct Eng* 140(2):04013058
- Vamvatsikos D, Cornell CA (2002) Incremental dynamic analysis. *Earthq Eng Struct Dyn* 31(3):491–514
- Yun S, Hamburger RO, Cornell CA, Foutch DA (2002) Seismic performance evaluation for steel moment frames. *J Struct Eng* 128(4):534. [https://doi.org/10.1061/\(asce\)0733-9445](https://doi.org/10.1061/(asce)0733-9445)

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